SECOND YEAR OF SURVEYING DEPARTMENT

THEORY OF ERROR 2017

LECTURE 3

RANDOM ERROR THEORY

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LECTURE ELEMENTS

1. "THEORY OF PROBABILITY"
2. "PROPERTIES OF THE NORMAL DISTRIBUTION CURVE"
3. "STANDARD NORMAL DISTRIBUTION FUNCTION"
4. "PROBABILITY OF THE STANDARD ERROR"
5. "USES FOR PERCENT ERRORS"
6. " PRACTICAL EXAMPLES "

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THEORY OF PROBABILITY

- PROBABILITY IS THE RATIO OF THE NUMBER OF TIMES THAT AN EVENT SHOULD OCCUR TO THE TOTAL NUMBER OF POSSIBILITIES.
- FOR EXAMPLE, THE PROBABILITY OF TOSSING A TWO WITH A FAIR DIE IS 1/6 SINCE THERE ARE SIX TOTAL POSSIBILITIES (FACES ON A DIE) AND ONLY ONE OF THESE IS A TWO.
- PROBABILITY IS ALWAYS A FRACTION RANGING BETWEEN ZERO AND ONE. ZERO DENOTES IMPOSSIBILITY, AND ONE INDICATES CERTAINTY.

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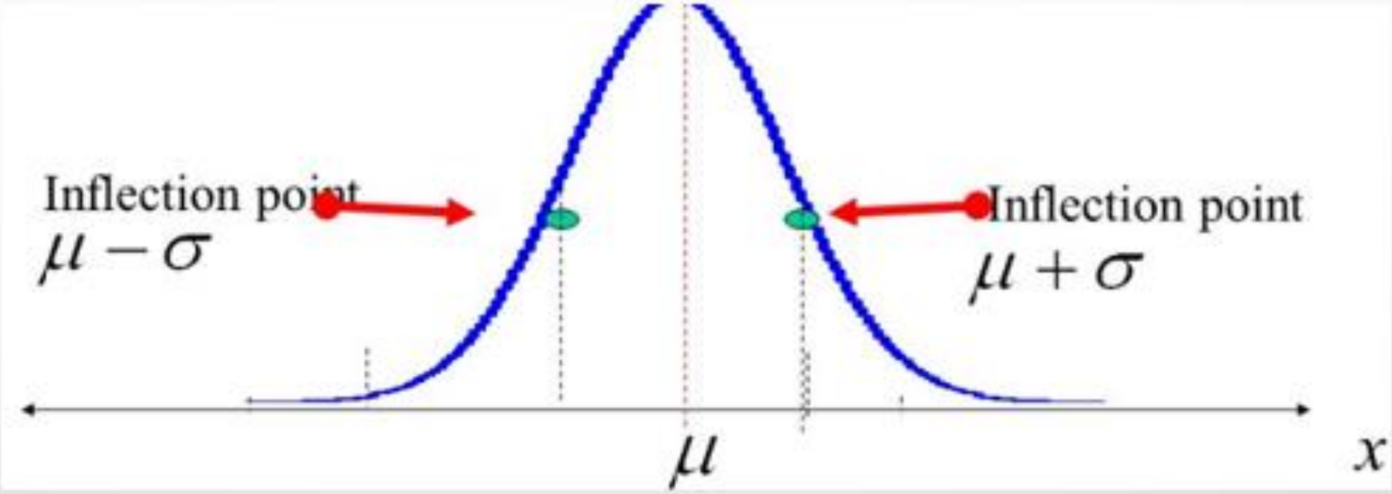
THEORY OF PROBABILITY

- A COMPOUND EVENT IS THE SIMULTANEOUS OCCURRENCE OF TWO OR MORE INDEPENDENT EVENTS. THIS IS THE SITUATION ENCOUNTERED MOST FREQUENTLY IN SURVEYING.
- FOR EXAMPLE, RANDOM ERRORS FROM ANGLES AND DISTANCES (COMPOUND EVENTS) CAUSE TRAVERSE MISCLOSURES. *THE PROBABILITY OF THE SIMULTANEOUS OCCURRENCE OF TWO INDEPENDENT EVENTS IS THE PRODUCT OF THEIR INDIVIDUAL PROBABILITIES*.

THEORY OF PROBABILITY

IF THE NUMBER OF COMBINING MEASUREMENTS, N, IS INCREASED PROGRESSIVELY TO LARGER VALUES, THE PLOT OF ERROR SIZES VERSUS PROBABILITIES WOULD APPROACH A SMOOTH CURVE OF THE CHARACTERISTIC BELL SHAPE SHOWN IN FIGURE 3.2. THIS CURVE IS KNOWN AS THE NORMAL ERROR DISTRIBUTION CURVE.



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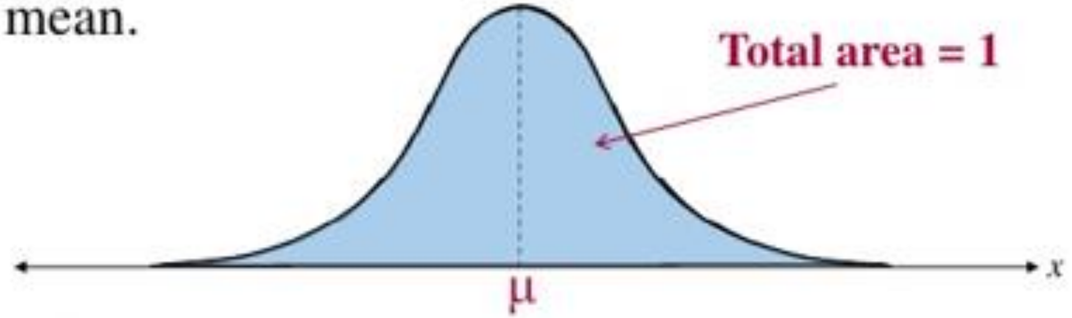
Properties of Normal Distributions

1. The mean, median, and mode are equal.
2. The normal curve is bell-shaped and symmetric about the mean.
3. The total area under the curve is equal to one.
4. The normal curve approaches, but never touches the x-axis as it extends farther and farther away from the mean.



THEORY OF PROBABILITY

- IT IS IMPORTANT TO NOTICE THAT THE TOTAL AREA OF THE VERTICAL BARS FOR EACH PLOT EQUALS 1.
- THIS IS TRUE NO MATTER THE VALUE OF N, AND THUS THE AREA UNDER THE SMOOTH NORMAL ERROR DISTRIBUTION CURVE IS EQUAL TO 1.
- IF AN EVENT HAS A PROBABILITY OF 1, IT IS CERTAIN TO OCCUR, AND THEREFORE THE AREA UNDER THE CURVE REPRESENTS THE SUM OF ALL THE PROBABILITIES OF THE OCCURRENCE OF ERRORS.

THEORY OF PROBABILITY

THE EQUATION OF THE NORMAL DISTRIBUTION CURVE, ALSO CALLED *THE NORMAL PROBABILITY DENSITY FUNCTION*, IS

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2}$$

WHERE *F*(X) IS THE PROBABILITY DENSITY FUNCTION, E THE BASE OF NATURAL LOGARITHMS, X THE ERROR, AND $\boldsymbol{\sigma}$ THE STANDARD ERROR

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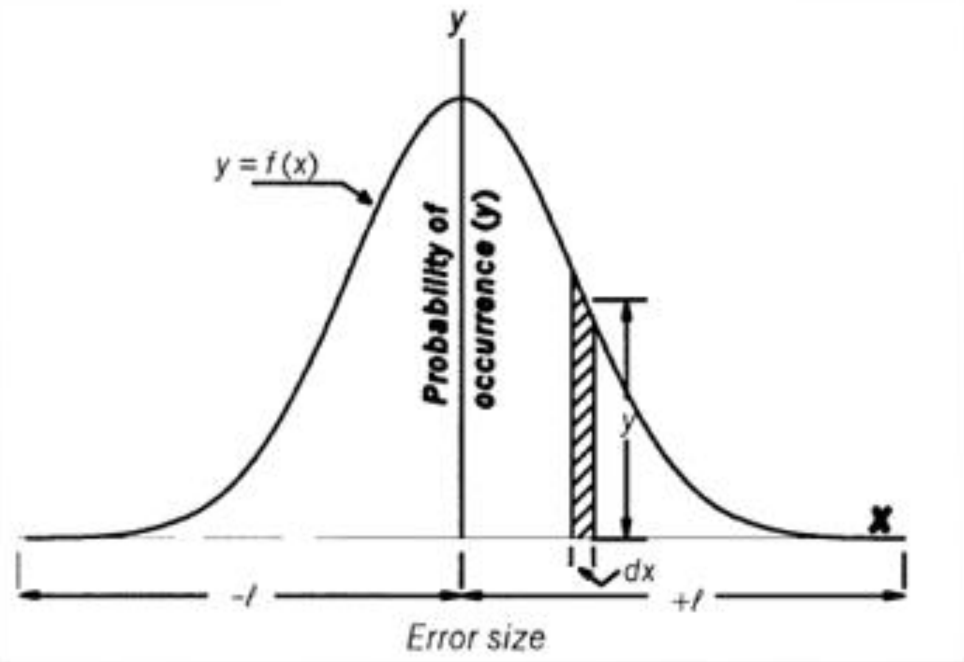
PROPERTIES OF THE NORMAL DISTRIBUTION CURVE

- *F*(X) IS THE PROBABILITY OF OCCURRENCE OF AN ERROR OF SIZE BETWEEN X AND X + DX, WHERE DX IS AN INFINITESIMALLY SMALL VALUE.
- THE ERROR'S PROBABILITY IS EQUIVALENT TO THE AREA UNDER THE CURVE BETWEEN THE LIMITS OF X AND X + DX, WHICH IS SHOWN CROSSHATCHED AS STATED PREVIOUSLY, THE TOTAL AREA UNDER THE PROBABILITY CURVE REPRESENTS THE TOTAL PROBABILITY, WHICH IS 1



$$\text{area} = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2}\, dx = 1$$

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PROPERTIES OF THE NORMAL DISTRIBUTION CURVE

- A FUNCTION'S SECOND DERIVATIVE PROVIDES THE RATE OF CHANGE IN A SLOPE WHEN EVALUATED AT A POINT.
- THE CURVE'S INFLECTION POINTS (POINTS WHERE THE ALGEBRAIC SIGN OF THE SLOPE CHANGES) CAN BE LOCATED BY FINDING WHERE THE FUNCTION'S SECOND DERIVATIVE EQUALS ZERO.
- IN EQUATION (3.8), D2Y/DX2 = 0 WHEN X2 /$\boldsymbol{\sigma}$2 -1 = 0, AND THUS THE CURVE'S INFLECTION POINT OCCURS WHEN X EQUALS $\pm\boldsymbol{\sigma}$.

$$y = \frac{1}{\sigma\sqrt{2\pi}}$$

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STANDARD NORMAL DISTRIBUTION FUNCTION

WE DEFINED THE PROBABILITY DENSITY FUNCTION OF A NORMAL RANDOM VARIABLE AS

$$f(x) = 1/(\sigma\sqrt{2\pi})e^{-x^2/2\sigma^2}$$

FROM THIS WE DEVELOP THE NORMAL DISTRIBUTION FUNCTION

$$F_x(t) = \int_{-\infty}^{t} \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2}\, dx$$

WHERE *T* IS THE UPPER BOUND OF INTEGRATION

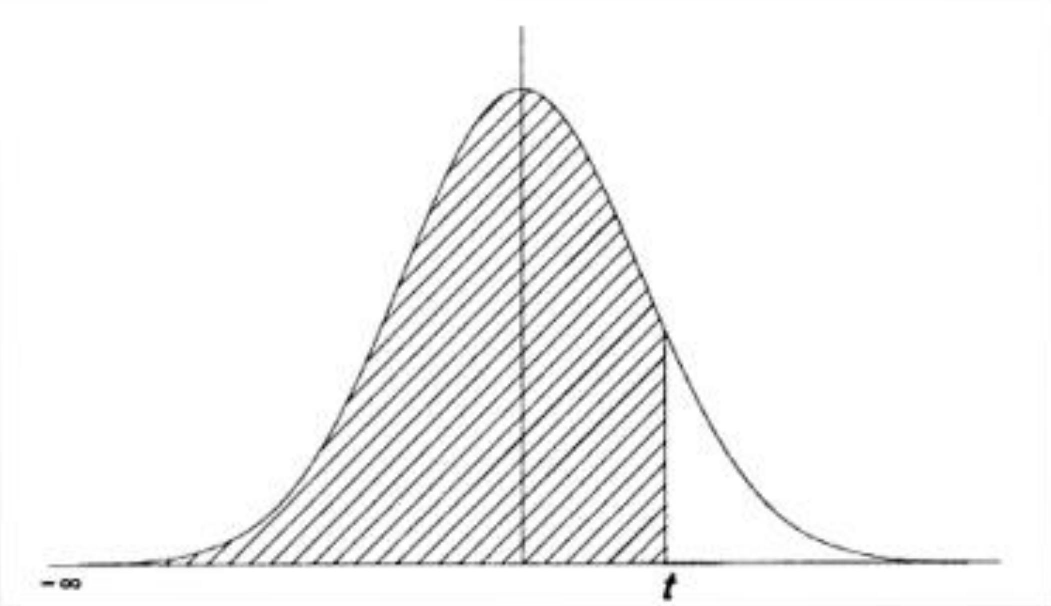
STANDARD NORMAL DISTRIBUTION FUNCTION

THIS HAS BEEN DONE FOR THE FUNCTION WHEN THE MEAN IS ZERO ($\mu = 0$) AND THE VARIANCE IS 1

($\boldsymbol{\sigma}2 = 1$).

THE RESULTS OF THIS INTEGRATION ARE SHOWN IN THE STANDARD NORMAL DISTRIBUTION TABLE D.1 REPRESENT AREAS UNDER THE STANDARD NORMAL DISTRIBUTION CURVE FROM -∞ TO T.

FOR EXAMPLE, TO DETERMINE THE AREA UNDER THE CURVE FROM -∞ TO 1.68, FIRST FIND THE ROW WITH 1.6 IN THE T COLUMN. THEN SCAN ALONG THE ROW TO THE COLUMN WITH A HEADING OF 8. AT THE INTERSECTION OF ROW 1.6 AND COLUMN 8 (1.68), THE VALUE 0.95352 OCCURS.



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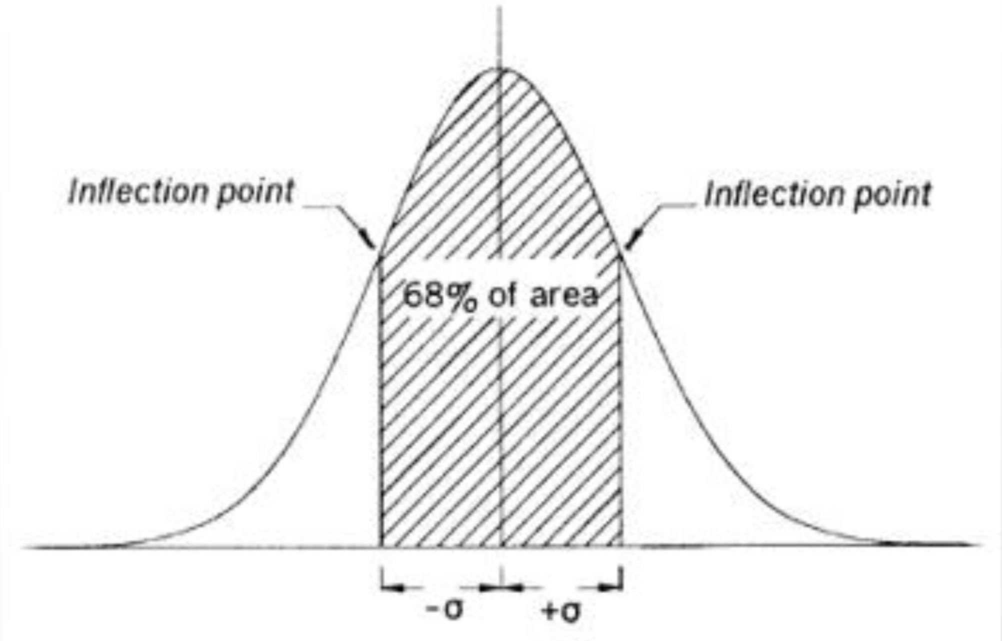
PROBABILITY OF THE STANDARD ERROR

FOR THE STANDARD NORMAL DISTRIBUTION WHEN

σ2 IS 1, IT IS NECESSARY TO LOCATE THE VALUES OF T = -1 (σ = -1) AND T = +1

(σ = 1) IN TABLE D.1.

$$P(-\sigma < z < +\sigma) = N_z(+\sigma) - N_z(-\sigma)$$
$$= 0.84134 - 0.15866 = 0.68268$$



FROM THIS IT HAS BEEN DETERMINED THAT ABOUT 68.3% OF ALL MEASUREMENTS FROM ANY DATA SET ARE EXPECTED TO LIE BETWEEN - σ AND + σ.

USES FOR PERCENT ERRORS

Multipliers for Various Percent Probable Errors

Symbol	Multiplier	Percent Probable Errors
E_{50}	0.6745σ	50
E_{90}	1.645σ	90
E_{95}	1.960σ	95
E_{99}	2.576σ	99
$E_{99.7}$	2.968σ	99.7
$E_{99.9}$	3.29σ	99.9

PRACTICAL EXAMPLES

- SUPPOSE THAT THE FOLLOWING VALUES (IN FEET) WERE OBTAINED IN 15 INDEPENDENT DISTANCE OBSERVATIONS, DI: 212.22, 212.25, 212.23, 212.15, 212.23, 212.11, 212.29, 212.34, 212.22, 212.24, 212.19, 212.25, 212.27, 212.20, AND 212.25. CALCULATE THE MEAN, S, E50, E95, AND CHECK FOR ANY OBSERVATIONS OUTSIDE THE 99.7% PROBABILITY LEVEL.

$$\overline{D} = \frac{\Sigma D_i}{n} = \frac{3183.34}{15} = 212.22 \text{ ft}$$

$$S = \sqrt{\frac{675{,}576.955 - 15(212.223^2)}{15 - 1}} = \sqrt{\frac{0.051298}{14}} = \pm 0.055 \text{ ft}$$

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PRACTICAL EXAMPLES

BY SCANNING THE DATA, IT IS SEEN THAT 10 OBSERVATIONS ARE BETWEEN 212.22 ± 0.06 FT OR WITHIN THE RANGE (212.16, 212.28).

THIS CORRESPONDS TO 10/15 * 100, OR 66.7% OF THE OBSERVATIONS. FOR THE SET, THIS IS WHAT IS EXPECTED IF IT ***CONFORMS TO NORMAL ERROR DISTRIBUTION THEORY***.

$$E_{50} = 0.6745S = \pm 0.6745\ (0.055) = \pm 0.04 \text{ ft}$$

AGAIN BY SCANNING THE DATA, NINE OBSERVATIONS LIE IN THE RANGE 212.22 ± 0.04 FT. THAT IS, THEY ARE WITHIN THE RANGE (212.18, 212.26) FT. THIS CORRESPONDS TO 9/15 * 100%, OR 60% OF THE OBSERVATIONS.

$$E_{95} = 1.960S = \pm 1.960(0.055) = \pm 0.11 \text{ ft}$$

NOTE THAT 14 OF THE OBSERVATIONS LIE IN THE RANGE 212.22 ± 0.11 (212.11, 212.33) FT, OR 93% OF THE DATA IS IN THE RANGE.

PRACTICAL EXAMPLES

AT THE 99.7% LEVEL OF CONFIDENCE, THE RANGE $\pm$ 2.968S CORRESPONDS TO AN INTERVAL OF $\pm$0.16 FT. WITH THIS CRITERION FOR REJECTION OF OUTLIERS, ALL VALUES IN THE DATA LIE IN THIS RANGE. THUS, THERE IS NO REASON TO BELIEVE THAT ANY OBSERVATION IS A BLUNDER OR OUTLIER.

REQUIRED REPORT

<u>PERFORM A REPORT COVERING THE FOLLOWING ITEMS</u>

1. STATE THE DIFFERENT RANDOM AND SYSTEMATIC ERRORS AT THE CADASTRAL MAPS PRODUCTION USING (TAPE SURVEYING) & TOTAL STATION, DISCUSS THE EQUATIONS OF THESE SYSTEMATIC ERRORS.
2. STATE THE DIFFERENT RANDOM AND SYSTEMATIC ERRORS AT THE TOPOGRAPHIC MAPS PRODUCTION USING LEVEL INSTRUMENT, DISCUSS THE EQUATIONS OF THESE SYSTEMATIC ERRORS.
3. IS IT BETTER TO USE TOTAL STATION OR TAPE OR LEVEL FOR MEASURING DISTANCES, WHY?
4. IS IT BETTER TO USE TOTAL STATION OR LEVEL FOR MEASURING HEIGHTS, WHY?

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THANKS

PLEASE DON'T USE THIS PRESENTATION WITHOUT GETTING A PERMEATION FROM ITS ORIGINAL OWNER

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